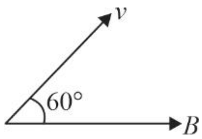
# Solutions of JEE Advanced - 1 | Paper - 1 | 2024

## PHYSICS

### SECTION – 1

**1.(BCD)** $\Rightarrow$ $\cos\theta = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$ $\Rightarrow$ $\boxed{\theta = 60^\circ}$

$\therefore$ The path will be helical.




$$v_\perp = \sqrt{2}v_0 \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{2}}v_0$$

$$v_{||} = \sqrt{2}v_0 \frac{1}{2} = \frac{v_0}{\sqrt{2}}$$

$$R = \frac{mv_\perp}{qB} = \sqrt{\frac{3}{2}}\frac{v_0}{\alpha\sqrt{2}B_0} = \frac{\sqrt{3}v_0}{2\alpha\beta_0}$$

$$T = \frac{2\pi m}{qB} = \frac{2\pi}{\alpha\sqrt{2}B_0} = \frac{\sqrt{2}\pi}{\alpha B_0}$$

$$P = v_{||} \times T \quad = \frac{v_0}{\sqrt{2}} \times \frac{\sqrt{2}\pi}{\alpha B_0} = \frac{\pi v_0}{\alpha B_0}$$

**2.(AC)** $\Rightarrow$ $\phi = \vec{B}\cdot\vec{A} \quad = \pi r^2 t^2$

$$\varepsilon = 2\pi r^2 t\,; \qquad i = \frac{2\pi r^2 t}{R}$$

$$\tau_{mag} = iAB \quad = \frac{2\pi r^2 t}{R}\pi r^2(2)$$

For toppling,

$$\tau_{mag} \geq mgr$$

$\therefore$ $\frac{4\pi^2 r^4 t}{R} \geq mgr\,;$ $\boxed{t \geq 1.25 s}$

$$H = \int i^2 R dt = \int \frac{4\pi^2 r^4 t^2}{R^2} R dt \quad = \frac{125}{24} J$$

**3.(BD)** The lower network of 4-8-6-2-4 is a wheat stone bridge in balanced condition since $4 \times 4 = 8 \times 2$. So, we remove $6\Omega$. Now, net resistance in series with battery is $6\Omega$.

$\Rightarrow$ Current through battery = 4A.

So, currents in the three branches, starting from the top are: $2A, \dfrac{2}{3}A, \dfrac{4}{3}A$

$\Rightarrow$ Ammeter reads $\dfrac{2}{3}A$

Voltmeter reading $= 24 - (1)(2) = 22V$

$\Rightarrow$ Power supplied $= 24 \times 4 = 96W$ $\Rightarrow$ Dissipation in $3\Omega = 2^2 \times 3 = 12W$

## SECTION-2

**4.(A)** Magnetic field $B = 1.5 \times 10^{-10} T$

Average intensity of the wave is given by

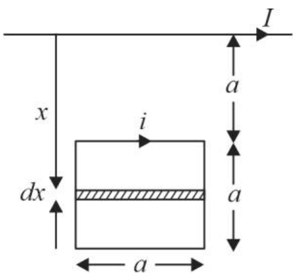
$$I_{av} = \frac{c}{\mu_0} B^2 = \left[\frac{(3.0 \times 10^8)(1.5 \times 10^{-10})^2}{4\pi \times 10^{-7}}\right] W/m^2$$

Hence, energy transported through the window

$$E = I_{av} \times A \times t = \frac{(3.0 \times 10^8)(1.5 \times 10^{-10})^2}{4\pi \times 10^{-7}} \times (0.20) \times (45) = 4.8 \times 10^{-5} J$$

**5.(B)** $\Rightarrow$ $\displaystyle\int d\phi = \int \frac{\mu_0 I}{2\pi x} a\, dx$; $\quad \phi = \dfrac{\mu_0 I a}{2\pi} \ell n 2$

Due to switching off the current, a clockwise current '$i$' will be induced.




$$i = \frac{d\phi}{Rdt} = \frac{\mu_0 a \ell n 2}{2\pi R} \frac{dI}{dt}$$

Now, force acting on loop is

$$F = F_1 - F_2 \quad = ia(B_1 - B_2)$$

$$= ia\left[\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{2\pi(2a)}\right]$$

$$F = \frac{\mu_0 i I}{4\pi} = \frac{\mu_0 I}{4\pi} \frac{\mu_0 a \ell n 2}{2\pi R} \frac{dI}{dt}; \qquad \int F dt = \frac{\mu_0^2 a \ell n 2}{8\pi^2 R} \int I\, dI$$

$$mv = \frac{\mu_0^2 a \ell n 2}{8\pi^2 R} \frac{I^2}{2}; \qquad v = \frac{(\mu_0 I)^2 a \ell n 2}{16\pi^2 R m}$$

**6.(A)** $F = eE$ ; $\quad a = \dfrac{e\sigma}{m\varepsilon_0}$

$$d = \frac{1}{2}at^2$$

$$d = \frac{e\sigma t^2}{2m\varepsilon_0} \quad \Rightarrow \quad t = \sqrt{\frac{2md\varepsilon_0}{\sigma e}}$$

$$R = ut \, ; \qquad \left[ R = u\sqrt{\frac{2m\varepsilon_0 d}{\sigma e}} \right]$$

**7.(B)** Motional emf will be induced along in the tangential direction and will get added up for the entire circumference.

$$E = Blv = B(2\pi r)v \, ; \quad i = \frac{E}{R}$$

## SECTION-3

**8.(B)**

**(A)** Sphere of radius $R$.

$$E = \frac{Q_{Total}}{4\pi\varepsilon_0 R^2} = \frac{\rho R}{3\varepsilon_0}; \quad V = \frac{3Q_{Total}}{8\pi\varepsilon_0 R} = \frac{\rho R^2}{2\varepsilon_0}$$

**(b)** Conducting shell

$$E = \frac{Q_{Total}}{4\pi\varepsilon_0 R^2} = \frac{\rho R}{3\varepsilon_0}; \quad V = V_{surface} = \frac{Q_{Total}}{4\pi\varepsilon_0 R} = \frac{\rho R^2}{3\varepsilon_0}$$

**(c)** Insulating shell of inner and outer radii $\dfrac{R}{2}$ and $R$

$$Q_{Total} = \frac{4\pi\rho}{3}\left(\frac{7R^3}{8}\right); \qquad E = \frac{Q_{Total}}{4\pi\varepsilon_0 R^2} = \frac{7R\rho}{24\varepsilon_0}$$

Charge enclosed inside spherical Gaussian surface of radius $x$.

$$q(x) = \frac{4\pi\rho}{3}\left(x^3 - \frac{R^3}{8}\right) \qquad \Rightarrow \qquad E(x) = \frac{\rho}{3x^2\varepsilon_0}\left(x^3 - \frac{R^3}{8}\right)$$

$$V_{surface} = \frac{Q_{Total}}{4\pi\varepsilon_0 R} = \frac{7\rho R^2}{24\varepsilon_0}$$

$$V_{centre} = V_{surface} + \int_{R/2}^{R} \frac{\rho}{3x^2\varepsilon_0}\left(r^3 - \frac{R^3}{8}\right)dx = \frac{7\rho R^2}{24\varepsilon_0} + \frac{\rho R^2}{12\varepsilon_0} = \frac{3\rho R^2}{8\varepsilon_0}$$

**(d)** $Q_{Total} = \displaystyle\int_0^R \frac{\rho r}{R}(4\pi r^2)dr = \rho\pi R^3$ **;** $\qquad E = \dfrac{Q_{Total}}{4\pi\varepsilon_0 R^2} = \dfrac{\rho R}{4\varepsilon_0}$

$$q(x) = \rho\pi x^3 \qquad \Rightarrow \qquad E(x) = \frac{\rho x}{4\varepsilon_0}$$

$$V_{centre} = V_{surface} + \int_0^R E(x)dx \quad = \frac{\rho R^2}{4\varepsilon_0} + \frac{\rho R^2}{8\varepsilon_0} = \frac{3\rho R^2}{8\varepsilon_0}$$

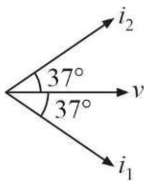
**9.(A)** **(a)** $Q_i = CV$ ; $Q_f = (2C)V = 2CV$

**(b)** $Q_i = \dfrac{CV}{2}$ ; $Q_f = \dfrac{2}{3}CV$

**(c)** $Q_i = CV$ ; $Q_f = 2CV$

**(d)** $Q_i = \dfrac{CV}{3}$ ; $Q_f = (2C)\left(\dfrac{V}{4}\right) = \dfrac{CV}{2}$

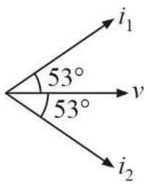
**10.(D)** (I)




$i_2$ leads $i_1$ by 74°

$\tan\phi = \dfrac{3}{4}$ ; $\phi = 37°$

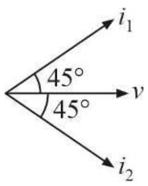
(II)




$i_1$ leads $i_2$ by 106°

$\tan\phi = \dfrac{4}{3}$ ; $\phi = 53°$

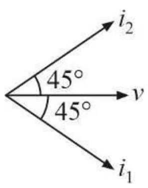
(III)




$i_1$ leads $i_2$ by 90°

$\tan\phi = 1$ ; $\phi = 45°$

(IV)



$i_2$ leads $i_1$ by 90°

$\tan\phi = 1$ ; $\phi = 45°$

**11.(A)** For a loop placed in uniform magnetic field,

$F = 0$; $\vec{\tau} = \vec{m} \times \vec{B}$

## SECTION-4

**1.(3)** Minimum separation will be when both the particles will have a common speed.

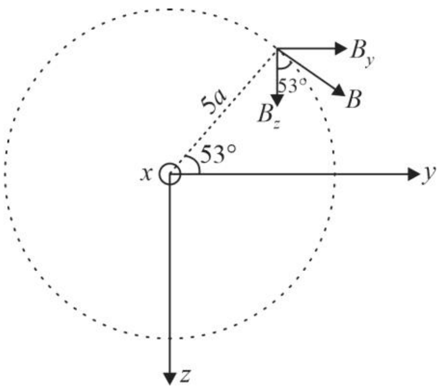
$p_i = p_f$

$\Rightarrow \quad mv_0 = (m + 2m)v$ ; $\quad v = \dfrac{v_0}{3}$

Also, $\quad K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}3m\frac{v_0^2}{9} + \frac{kq^2}{r}; \qquad r = \frac{3kq^2}{mv_0^2}$$

**2.(7)** $B = \dfrac{\mu_0 i}{2\pi(5a)}$




$$\vec{B} = By\hat{j} + Bz\hat{k} = \frac{4}{5}B\hat{j} + \frac{3}{5}B\hat{k}$$

$$= \frac{B}{5}(4\hat{j} + 3\hat{k}) = \frac{\mu_0 i}{50\pi a}(4\hat{j} + 3\hat{k})$$

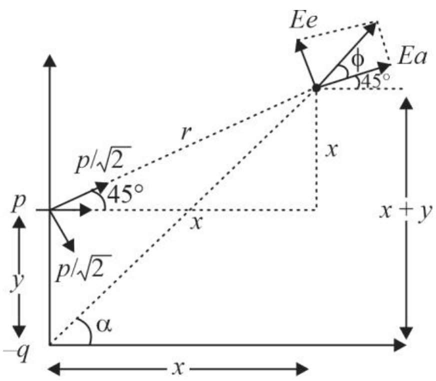
$\therefore \quad p + q = 7$

**3.(2)** $U_i = \dfrac{Q^2}{2C} = \dfrac{Q^2 d}{2\varepsilon_0 A}$; $\qquad U_f = \dfrac{Q^2 3d}{2\varepsilon_0 A}$

$$W_{ext} = u_f - u_i = \frac{Q^2 d}{\varepsilon_0 A} \qquad = \left(\frac{\varepsilon_0 A}{d}V\right)^2 \frac{d}{\varepsilon_0 A} = \frac{\varepsilon_0 A V^2}{d}$$

$\therefore \quad N = 2$

**4.(2)** $Ea = \dfrac{2kp/\sqrt{2}}{r^3}$; $\quad Ee = \dfrac{kp/\sqrt{2}}{r^3}$




$\tan\phi = \dfrac{1}{2}$; $\quad \tan\alpha = \tan(45+\phi)$

$$\frac{x+y}{x} = \frac{1+\tan\phi}{1-\tan\phi} = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}}$$

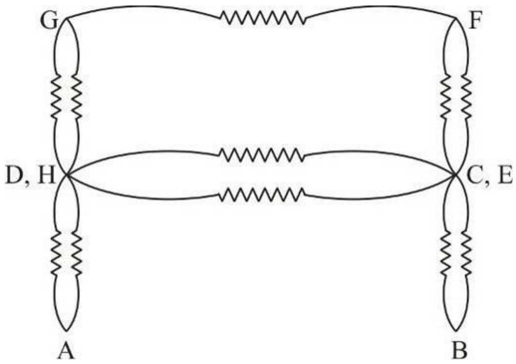
$1+\dfrac{y}{x} = 3$; $\quad y = 2x$

**5.(4)** From the graph,

$$V = \begin{cases} 4 & ; \quad 0 \le t \le 1 \\ -4t+4 & ; \quad 1 < t \le 2 \end{cases}$$

$$\Rightarrow \quad V_{rms} = \sqrt{\frac{\int_0^1 (4)^2 dt + \int_1^2 (-4t+4)^2 dt}{2-0}} = \sqrt{\frac{16+\frac{16}{3}}{2}} = 4\sqrt{\frac{2}{3}}V$$

**6.(7)** Seeing plane of symmetry AGEB, circuit can be redrawn like this



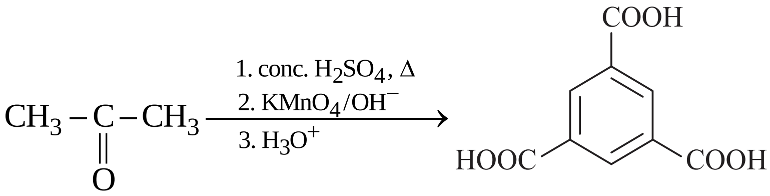

Solve to get $R_{AB} = \dfrac{7R}{5}$

# CHEMISTRY

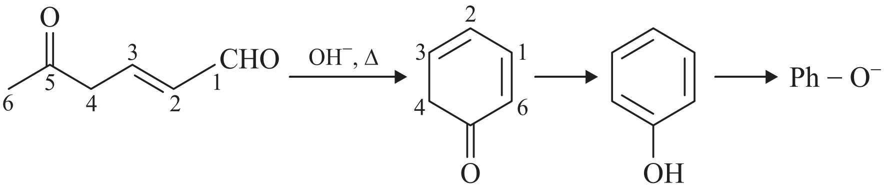
## SECTION – 1

**1.(ABCD)**

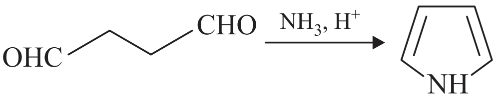
**(A)**



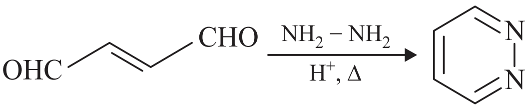

**(B)**



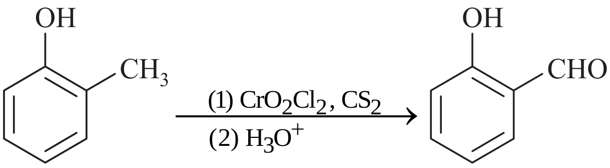

**(C)**



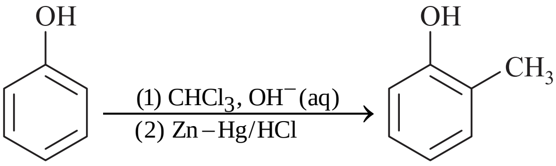

**(D)**



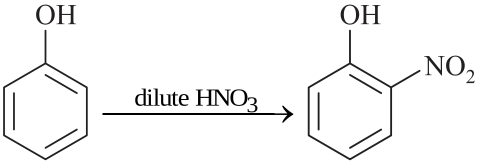

**2.(BC)** **(A)**



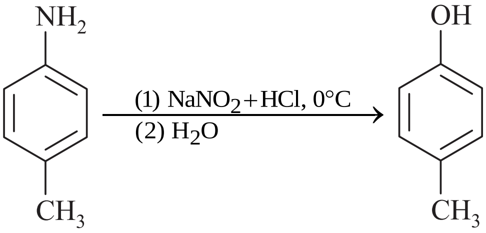

**(B)**




**(C)**




**(D)**



**3.(ABD)**

| Positively charged sols | Negatively charged sols |
|---|---|
| Hydrated metallic oxides, e.g., $Al_2O_3 \cdot xH_2O$, $CrO_3 \cdot xH_2O$ and $Fe_2O_3 \cdot xH_2O$, etc. | Metals e.g., copper, silver, gold sols. |
| Basic dye stuffs, e.g., methylene blue sol. | Metallic sulphides, e.g., $As_2S_3$, $Sb_2S_3$, CdS sols. |
| | Acid dye stuffs, e.g., eosin, congo red sols. |
| | Sols of starch, gum, gelatin, clay, charcoal, etc. |

As gelatin is negatively charged thus it can't prevent flocculation of positively charged hydrated metal oxides sols.

## SECTION-2

**4.(C)** (I) $\rightarrow$ 3D hcp, coordination number = 12, number of spheres containing their 100% volume is zero.

(II) $\rightarrow$ ccp, coordination number = 12

Gap between identical layers (say A) is $\frac{4R\sqrt{2}}{\sqrt{3}}$ for (I) and $\frac{4R\sqrt{3}}{\sqrt{2}}$ for (II).

**5.(D)** $\pi = C \times \frac{RT}{m_B} \times 1000$

Where C = concentration in g/cc.

Slope $= \frac{RT}{m_B} \times 1000$

$$m_B = \frac{RT}{Slope} \times 1000 = \frac{0.0821 \times 293 \times 1000}{4.65 \times 10^{-3}} = 5.17 \times 10^6$$

**6.(C)** $\frac{d[B]}{dt} = k_1[A]_t - k_2[B]_t =$ Net r. o. f. of B

$\frac{d[C]}{dt} = k_2[B]_t =$ r. o. f of C

As $\quad k_1[A]_t - k_2[B]_t = 9k_2[B]_t$

$$k_1[A]_t = 10k_2[B]_t$$

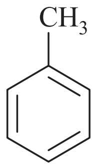
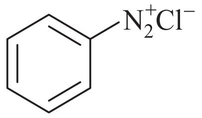
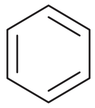
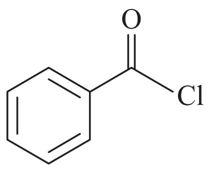
$$[B]_t = \frac{k_1}{k_2}\frac{[A]_t}{10} = \frac{[A]_t}{5}$$

**7.(B)** $\Delta G^\circ = -nFE^\circ_{cell} = W_{(\text{Max non PV})}$

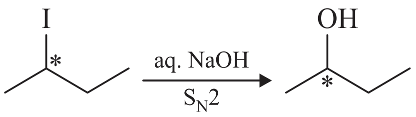
$$\Delta G^\circ = -2F[+0.02+1.50] = -2F[1.52] = -3.04F$$

## SECTION-3

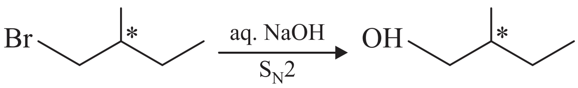
**8.(D)**

| Reaction from List-I | | Reactant | Correct match from List-II |
|---|---|---|---|
| **(P)** | Etard reaction |  | 3 |
| **(Q)** | Gattermann reaction |  | 4 |
| **(R)** | Gattermann-Koch reaction |  | 5 |
| **(S)** | Rosenmund reaction |  | 2 |

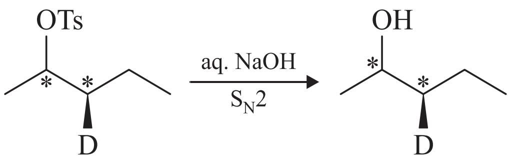
**9.(A)**



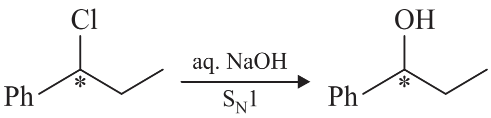

(Single enantiomer of inverted configuration)




(Single enantiomer with retention of configuration)




(A mixture of diastereomers)




(Racemic mixture)

**10.(B)**

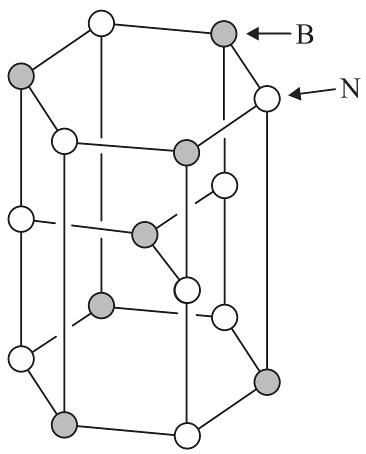
| | | Anode Product | Cathode Product |
|---|---|---|---|
| **(1)** | $CuSO_4(aq)$ electrolysis using Pt electrodes | $O_2(g)$ | $Cu(s)$ |
| **(2)** | Dilute $H_2SO_4$ electrolysis using Pt electrodes | $O_2(g)$ | $H_2(g)$ |
| **(3)** | Electrolysis of conc. NaCl using Pt electrodes | $Cl_2(g)$ | $H_2(g)$ |
| **(4)** | Electrolysis of NaCl(aq) using Hg cathode and graphite anodes | $Cl_2(g)$ | $Na-Hg$ |
| **(5)** | Electrolysis of $CH_3COONa(aq)$ using Pt electrodes | $CH_3-CH_3(g)$, $CO_2(g)$ | $H_2(g)$ |

**11.(B)**

| | **Relation between X and Y** |
|---|---|
| $X \longrightarrow 2\,{}^{4}_{2}\alpha + 4\,{}^{0}_{-1}\beta + Y$ | Y is isotope of X |
| $X \longrightarrow {}^{4}_{2}\alpha + Y$ | Y is isodiaphere to X |
| $X \longrightarrow {}^{0}_{-1}\beta + Y$ | Y is isobar to X |
| $X \longrightarrow {}^{0}_{+1}\beta + Y$ | Y is isobar to X |
| $X \longrightarrow {}^{1}_{+1}P + Y$ | Y is isotone to X |

**SECTION-4**

**1.(2)**

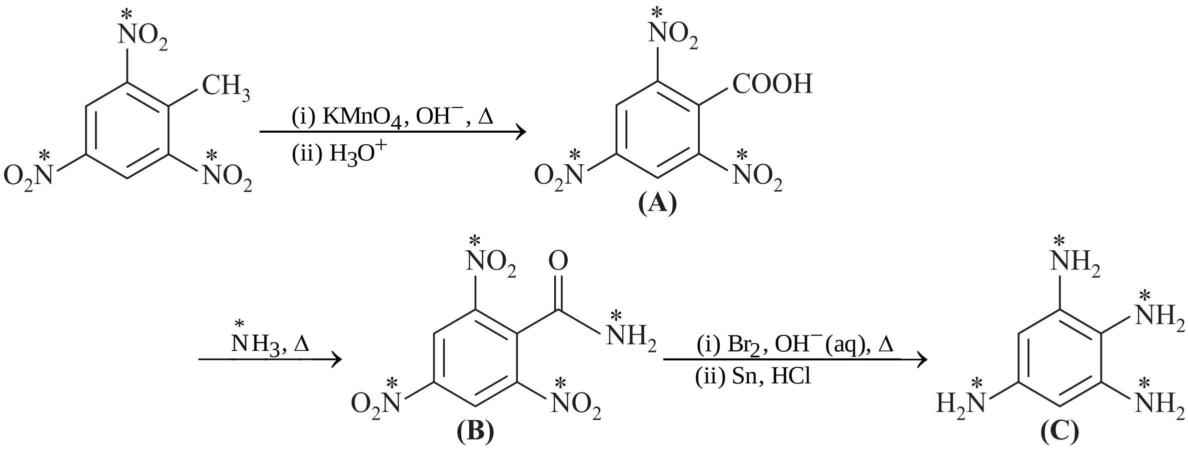



Following is the arrangement of atoms in solid graphite and BN(s).

Effective number of B atoms $= 6\times\frac{1}{6}+1\times1 = 2$

Effective number of N atoms $= 6\times\frac{1}{6}+3\times\frac{1}{3} = 2$ $\therefore$ Number of BN formula units = 2 per unit cell.

**2.(42)**



Mass percentage of N in compound $C = \frac{15\times4\times100}{15\times4+6\times12+10\times1} = \frac{60\times100}{60+72+10} = \frac{6000}{142} = 42.25$

**3.(3)**

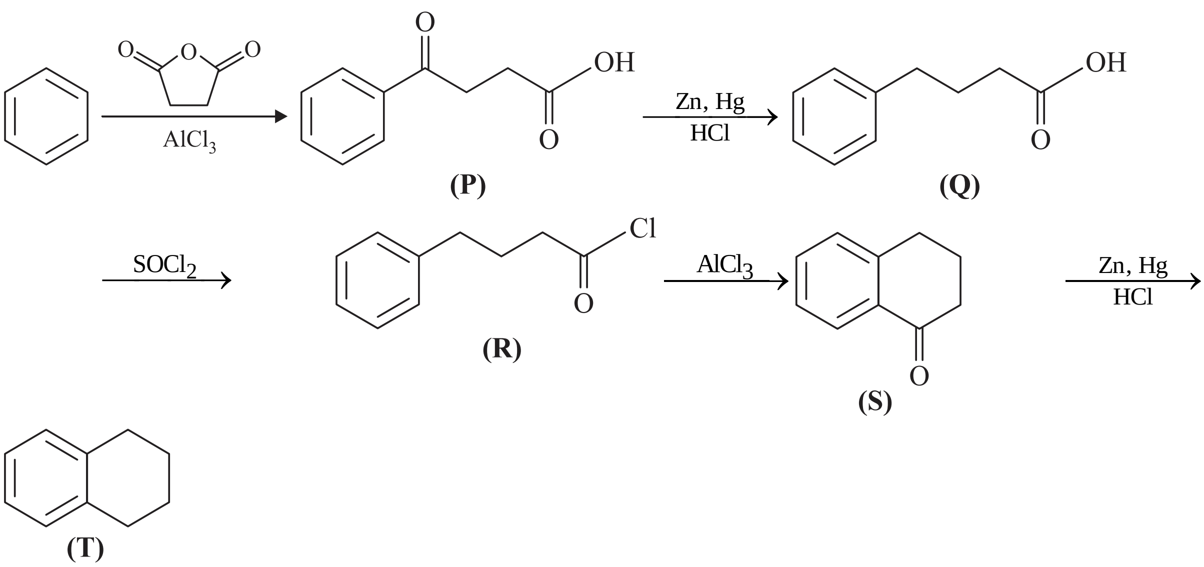
| | $2N_2O_5 \xrightarrow{k}$ | $2NO_2$ | $+\ O_2$ |
|---|---|---|---|
| $t = 0$ | 100 | 0 | 0 |
| $t = 100$ | 80 | 20 | 10 |

As $\quad 2k = \frac{2.303}{50}\log_{10}\frac{(P_{N_2O_5})_{t=0}}{(P_{N_2O_5})_t}$

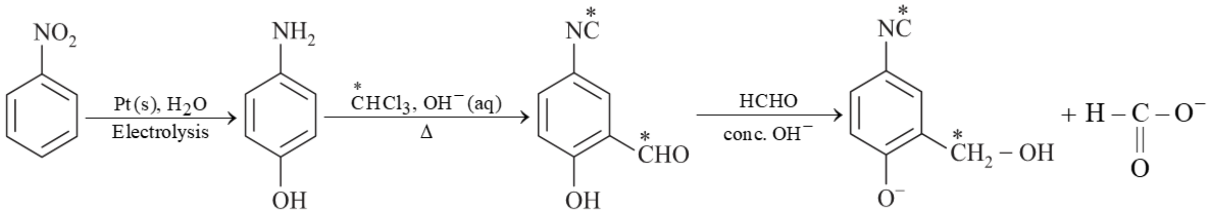
$$2k = \frac{2.303}{50}(\log_{10}5-\log 4)$$

$$k = \frac{2.303}{100}[0.70-0.60] = 2.303\times10^{-3}\,\text{min}^{-1}$$

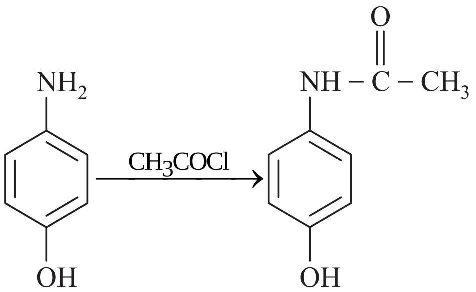
**4.(5)**



**5.(2)**




(P) (Q) (R)




**(P)** **(Paracetamol)**

**6.(1)** $E°$ for reaction 1 $= E^{\circ}_{\frac{1}{2}Cl_2/Cl^-} - E^{\circ}_{ClO^-/\frac{1}{2}Cl_2} = 1.20$

$$\Rightarrow \quad E^{\circ}_{ClO^-/\frac{1}{2}Cl_2} = 1.35 - 1.20 = 0.15\,\text{volt}$$

Now as $E^{\circ}_{ClO^-/\frac{1}{2}Cl_2} + E^{\circ}_{\frac{1}{2}Cl_2/Cl^-} = 2E^{\circ}_{ClO^-/Cl^-}$ [Using the reaction sequence: $ClO^- \longrightarrow \frac{1}{2}Cl_2 \longrightarrow Cl^-$]

$$E^{\circ}_{ClO^-/Cl^-} = \frac{(0.15+1.35)}{2} = 0.75\,\text{volt}$$

$E°$ for reaction 2 $= E^{\circ}_{ClO^-/Cl^-} - E^{\circ}_{\text{Acetone/Propan-2-ol}} = E^{\circ}_{ClO^-/Cl^-} - (-0.25)$

$$= E^{\circ}_{ClO^-/Cl^-} + 0.25 = 1\,\text{volt}$$

# MATHEMATICS

## SECTION – 1

**1.(ABD)**

$$f(0) = max\{1+sin0, 1, 1-cos0\} = 1$$

$$g(0) = max\{1, |0-1|\} = 1$$

$$f(1) = max\{1+sin1, 1, 1-cos1\} = 1+sin1$$

$$g(f(0)) = g(1) = max\{1, |1-1|\} = 1$$

$$f(g(0)) = f(1) = 1+sin1$$

$$g(f(1)) = g(1+sin1) = max\{1, |1+sin1-1|\} = 1$$

**2.(ABC)**

Since, $\lim_{x\to1^-} g(x) = \lim_{x\to1^+} g(x) = 1$ and $g(1) = 0$.

So, $g$ ($x$) is not continuous at $x$ = 1 but $\lim_{x\to1} g(x)$ exists.

We have $\lim_{x\to1^-} f(x) = \lim_{h\to1} f(1-h) = \lim_{x\to1}[1-h] = 0$

and $\lim_{x\to1^+} f(x) = \lim_{h\to0} f(1+h) = \lim_{h\to0}[1+h] = 1$

So, $\lim_{x\to1} f(x) = 0$ does not exist hence f (x) is not continuous at $x$ = 1

We have $gof(x) = g(f(x)) = g([x]) = 0, \forall\, x \in R$

So, $gof$ is continuous for all $x$.

We have $fog(x) = f(g(x)) = \begin{cases} f(0), & x \in Z \\ f(x^2), & x \in R-Z \end{cases} = \begin{cases} 0, & x \in Z \\ [x^2], & x \in R-Z \end{cases}$

Which is clearly not continuous

**3.(AB)** $f(x) = e^x + \int_0^1 e^x f(t)\,dt = e^x + ke^x$ where $k = \int_0^1 f(t)\,dt$

$\therefore \quad k = \int_0^1 \left(e^t + ke^t\right)dt = e + ke - 1 - k$

$\therefore \quad k = \dfrac{e-1}{2-e}$, thus $f(x) = e^x\left(1 + \dfrac{e-1}{2-e}\right) = \dfrac{e^x}{2-e}$

Obviously, $f(0)=\dfrac{1}{2-e}<0$

Also, $f'(x)=\dfrac{1e^x}{2-e}<0$ for $\forall\ x\in R$

Hence, $f(x)$ is a decreasing function.

Also $\displaystyle\int_0^1 f(x)\,dx=\int_0^1\frac{e^x}{2-e}\,dx$

$$=\left[\frac{e^x}{2-e}\right]_0^1=\frac{e-1}{2-e}<0$$

**SECTION-2**

**4.(D)** $\displaystyle\sum_{m=1}^{n}\tan^{-1}\left[\frac{2m}{1+\left(m^2+m+1\right)\cdot\left(m^2-m+1\right)}\right]$

$$=\sum_{m=1}^{n}(\tan^{-1}(m^2+m+1)-\tan^{-1}(m^2-m+1))=\tan^{-1}(n^2+n+1)-\tan^{-1}(1)=\tan^{-1}\left(\frac{n^2+n}{2+n+n^2}\right)$$

**5.(C)** $y=(\sin^{-1}x)^3+\left(\dfrac{\pi}{2}-\sin^{-1}x\right)^3$ ; Let $t=\sin^{-1}x$ ; $-\dfrac{\pi}{2}\le t\le\dfrac{\pi}{2}$ ; $y=t^3+\dfrac{\pi^3}{8}-t^3-\dfrac{3\pi}{2}.t\left(\dfrac{\pi}{2}-t\right)$

$y=\dfrac{3\pi}{2}t^2-\dfrac{3\pi^2}{4}.t+\dfrac{\pi^3}{8}$ ; $y'=3\pi t-\dfrac{3\pi^2}{4}=0 \quad\Rightarrow\quad t=\dfrac{\pi}{4}$

At $t=\dfrac{\pi}{4};y=\left(\dfrac{\pi}{4}\right)^3+\left(\dfrac{\pi}{2}-\dfrac{\pi}{4}\right)^3=\dfrac{\pi^3}{32}$ ; at $t=-\dfrac{\pi}{2};y=\left(-\dfrac{\pi}{2}\right)^3+\left(\dfrac{\pi}{2}+\dfrac{\pi}{2}\right)^3=\pi^3-\dfrac{\pi^3}{8}=\dfrac{7\pi^3}{8}$

At $t=\dfrac{\pi}{2}$ ; $y=\dfrac{\pi^3}{8}$ $\quad\therefore$ min. value $=\dfrac{\pi^3}{32}$ ; max. value $=\dfrac{7\pi^3}{8}$

**6.(D)** Let two observations ae $x$ and $y$.

$$\bar{x}=\frac{5+7+10+12+14+15+x+y}{8}$$

$$10=\frac{63+x+y}{8}\Rightarrow x+y=17 \qquad \textbf{....(i)}$$

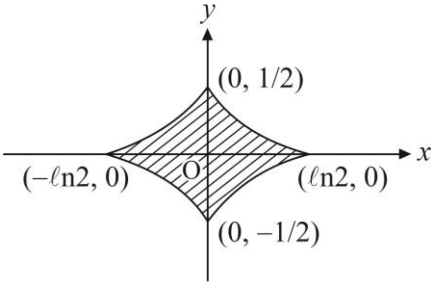
$$var(x)=13.5=\left(\frac{25+49+100+144+196+225+x^2+y^2}{8}\right)-\left(10^2\right)$$

$$\Rightarrow \quad x^2+y^2=169 \qquad \textbf{....(ii)}$$

From (i) and (ii) $(x, y) = (12, 5)$ or $(5, 12)$

Therefore, the absolute difference of the two observation = $|12-5|$ or $|5-12| = 7$

**7.(D)** Area $4\int_0^{\ell n2}\left(e^{-x}-\frac{1}{2}\right)dx = 2-2\ell n2$




**SECTION-3**

**8.(A)** **[A-R] [B-S] [C-P] [D-Q]**

$[x]+\{x\}+[y]+\{z\}=12.7$ **. . . .(i)**

$[x]+\{y\}+[z]+\{z\}=4.1$ **. . . .(ii)**

$\{x\}+[y]+\{y\}+[z]=2$ **. . . .(iii)**

Adding (i), (ii) and (iii),

$\Rightarrow \quad [x]+\{x\}+[y]+\{y\}+[z]+\{z\}=9.4$

$\Rightarrow \quad \{y\}+[z]=-3.3, \{x\}+[y]=5.3, [x]+\{z\}=7.4$

$\Rightarrow \quad \{y\}=0.7, [z]=-4, \{x\}=0.3, [y]=5$

$[x]=7, \{z\}=0.4$

**9.(D)** **[A-S] [B-Q] [C-P] [D-R]**

**(A)** $\int\frac{dx}{\left(x^2+1\right)\sqrt{x^2+2}}=\int\frac{\frac{1}{x^3}}{\left(1+\frac{1}{x^2}\right)\sqrt{1+\frac{2}{x^2}}}\,dx$

Let $1+\frac{2}{x^2}=t^2$

**(B)** $x+2=\frac{1}{t}$

**(C)** $\int\frac{x^4+x^8}{\left(1-x^4\right)^{7/2}}dx=\int\frac{\left(x+\frac{1}{x^3}\right)dx}{\left(\frac{1}{x^2}-x^2\right)^{7/2}}$

Let $\frac{1}{x^2} - x^2 = t^2$

**(D)** $\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} = \frac{1-\sqrt{x}}{\sqrt{1-x}}$

**10.(B)** [A-R] [B-Q] [C-S] [D-P]

**(A)** Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\int_0^1 \frac{dt}{(1+t)(2+t)} = \int_0^1 \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt = \left[\ell n(1+t) - \ell n(t+2)\right]_0^1$$

**(B)** $$\int_0^{41\pi/4} |\cos x|\, dx = 10\int_0^{\pi} |\cos dx|\, dx + \int_0^{\pi/4} \cos x\, dx$$

$$= \left[\int_0^{\pi/2} \cos x\, dx - \int_{\pi/2}^{\pi} \cos x\, dx\right] + \int_0^{\pi/4} \cos x\, dx$$

**(C)** $$\int_{-1/2}^{0} [x]\, dx + \int_0^{1/2} [x]\, dx + \int_{-1/2}^{1/2} \ell n\left(\frac{1+x}{1-x}\right) dx = \int_{-1/2}^{0} -1\, dx + \int_0^{1/2} 0\, dx = -\frac{1}{2}$$

**(D)** $$I = \int_0^{\pi/2} \frac{2\sqrt{\cos\theta}}{3\left(\sqrt{\cos\theta} + \sqrt{\sin\theta}\right)} d\theta = \int_0^{\pi/2} \frac{2\sqrt{\sin\theta}}{3\left(\sqrt{\sin\theta} + \sqrt{\cos\theta}\right)} d\theta$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{2}{3} d\theta = \frac{\pi}{3} \quad \Rightarrow \quad I = \frac{\pi}{6}$$

**11.(C)** [A-P,Q,R, S] [B-R] [C-P] [D-P]

**(A)** $$f(x) = \frac{1}{\frac{1}{(x-1)^2} + \frac{1}{x-1} + 2}$$

$$= \frac{(x-1)^2}{1 + x - 1 - 2(x-1)^2} = \frac{-(x-1)^2}{2x^2 - 5x + 2} = \frac{-(x-1)^2}{(x-2)(2x-1)}$$

So, P, Q, R, S are correct

**(B)** $f(x) = \operatorname{sgn} x\left(1 - x^2\right)$

So, R is correct as it is discontinuous at $x = 0, \pm 1$

So, R is correct

**(C)** $\lim_{x\to 0} \frac{f(x)}{x}$ exists and $f(0) = 0$

$\Rightarrow$ $f(x)$ is continuous at $x = 0$

So, P is correct

**(D)** $f\left(x\right)=\lim\limits_{n\to\infty} x+\dfrac{\frac{\text{sgn}\, x}{n}}{1+\frac{1}{n}}=x$

So, P is correct

## SECTION-4

**1.(7)** Let $2x+y=3x-y \Rightarrow 2y=x \Rightarrow y=\dfrac{x}{2}$

$\therefore$ Put $y=\dfrac{x}{2}$

$\Rightarrow$ $f\left(x\right)+f\left(\dfrac{5x}{2}\right)+\dfrac{5x^2}{2}=f\left(\dfrac{5x}{2}\right)+2x^2+1$

$\Rightarrow$ $f\left(x\right)=1-\dfrac{x^2}{2}$

$\Rightarrow$ $f\left(4\right)=-7$

**2.(8)** Since RHS is finite quantity

$\therefore$ At $x\to1$, Numerator must be = 0

$\therefore$ $0 + b + 4 = 0$

$\therefore$ $b=-4$

Then $\lim\limits_{x\to1}\dfrac{a\,sin\left(x-1\right)-4\,cos\left(x-1\right)+4}{\left(x^2-1\right)}=-2$

Put $x=1+h$, Then $\lim\limits_{h\to0}\dfrac{a\,sinh+4\left(1-cos\,h\right)}{h\left(2+h\right)}=-2$

$\Rightarrow$ $\lim\limits_{h\to0}\dfrac{a\left(\frac{sinh}{h}\right)+4\left(\frac{1-cosh}{h}\right)}{2+h}=-2$

$\Rightarrow$ $\dfrac{a\left(1\right)+0}{2}=-2$

$\Rightarrow$ $a=-4$

$\Rightarrow$ $\left|a+b\right|=8$

**3.(8)** $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{f(x) + f(h) + 2xh(x+h) - \frac{1}{3} - \left(f(x) + f(0) - \frac{1}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{f(h) - f(0)}{h} + 2x^2 = f'(0) + 2x^2$$

$$\lim_{h\to 0} \frac{3f(h)-1}{6h} = \lim_{h\to 0} \frac{f(h) - \frac{1}{3}}{2h} = \lim_{h\to 0} \frac{f(h) - f(0)}{2h}$$

$$= \frac{f'(0)}{2} = \frac{2}{3} \Rightarrow f'(0) = \frac{4}{3}$$

$\therefore \quad f'(x) = \dfrac{4}{3} + 2x^2$

$$f(x) = \lambda + \frac{4}{3}x + \frac{2x^3}{3} \Rightarrow f(0) = \lambda = \frac{1}{3}$$

$\therefore \quad f(x) = \dfrac{2x^3}{3} + \dfrac{4}{3}x + \dfrac{1}{3} \Rightarrow f(2) = \dfrac{25}{3}$

Therefore, the value of $\left[f(2)\right] = \left[\dfrac{25}{3}\right] = 8$

**4.(5)** We have $(gof)(x) = x$

$\Rightarrow \quad g'(f(x))f'(x) = 1$

when $\quad f(x) = -\dfrac{7}{6} \Rightarrow x = 1$

$\Rightarrow \quad g'\left(-\dfrac{7}{6}\right)f'(1) = 1$

Hence $\quad g'\left(-\dfrac{7}{6}\right) = \dfrac{1}{f'(1)} = \dfrac{1}{5}$

**5.(3)** $\dfrac{dy}{dx} = \dfrac{y}{x} = -\dfrac{1}{2}cot^3\theta = -\dfrac{1}{2}$ at $\theta = \dfrac{\pi}{4}$

Also the point P for $\theta = \dfrac{\pi}{4}$ is (2, 1)

Equation of tangent is $y - 1 = -\dfrac{1}{2}(x-2)$

or $x+2y-4=0$ ....**(i)**

This meets the curve whose Cartesian equation on eliminating $\theta$ by $sec^2\theta - tan^2\theta = 1$ is

$$y^2 = \frac{1}{x-1} \qquad \text{....(ii)}$$

Solving (i) and (ii), we get :

$$y = 1,\ -\frac{1}{2}$$

$\therefore$ $x = 2\ 5$

Hence P is (2, 1) as given and Q is $\left(5, -\frac{1}{2}\right)$

$\therefore$ $PQ = \sqrt{\frac{45}{4}} = \frac{3\sqrt{5}}{2}$

**6.(3)** $f''(x) = 4x$

$$f'(x) = 2x^2 + C$$

Given $f'(-2) = 1 \Rightarrow C = -7$

$\therefore$ $f'(x) = 2x^2 - 7$

$$f(x) = \frac{2}{3}x^3 - 7x + C,\ f(-2) = 0$$

$$0 = -\frac{16}{3} + 14 + C \Rightarrow C = -\frac{26}{3}$$

$\therefore$ $f(x) = \frac{2}{3}x^3 - 7x - \frac{26}{3} = \frac{1}{3}\left(2x^3 - 21x - 26\right)$

$\therefore$ $f(1) = -15$

Therefore, the value of $\left[\frac{f(1)}{5}\right] = \left|-\frac{15}{5}\right| = 3$